

1. Factor each of the following by grouping. You might have to reorder the terms before you can factor by grouping. Check your answers by multiplying all the factors back together.

(a) $\overbrace{ab + b} + \overbrace{2a + 2}$

$$= b(a+1) + 2(a+1)$$

$$= (b+2)(a+1)$$

check:

$$(b+2)(a+1)$$

$$= ab + b + 2a + 2$$

(b) $\overbrace{x^3 - x^2} - \overbrace{3x + 3}$

$$= x^2(x-1) - 3(x-1)$$

$$= (x^2-3)(x-1)$$

check:

$$(x^2-3)(x-1)$$

$$= x^3 - x^2 - 3x + 3$$

(c) $\overbrace{6ax - by + 2bx - 3ay}$

$$= 3a(2x-y) + b(-y+2x)$$

$$= (3a+b)(2x-y)$$

check:

$$(3a+b)(2x-y)$$

$$= 6ax - 3ay + 2bx - by$$

(d) $\overbrace{2rs + 4s} - \overbrace{r - 2}$

$$= 2s(r+2) - (r+2)$$

$$= (2s-1)(r+2)$$

check:

$$(2s-1)(r+2) = 2rs + 4s - r - 2$$

(e) $\overbrace{x^2y + 6x - 3x^3 - 2y}$

$$= y(x^2-2) + 3x(2-x^2)$$

$$= y(x^2-2) - 3x(x^2-2)$$

$$= (y-3x)(x^2-2)$$

check:

$$(y-3x)(x^2-2)$$

$$= x^2y - 2y - 3x^3 + 6x$$

2. Factor each of the following trinomials. Check your answers by multiplying all the factors back together.

$$\begin{aligned} \text{(a)} \quad & x^2 - 12x + 32 \\ & = (x - 8)(x - 4) \end{aligned}$$

check:

$$\begin{aligned} (x - 8)(x - 4) &= x^2 - 4x - 8x + 32 \\ &= x^2 - 12x + 32 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & y^2 + 10y - 24 \\ & = (y + 12)(y - 2) \end{aligned}$$

check:

$$\begin{aligned} (y + 12)(y - 2) &= y^2 + 12y - 2y - 24 \\ &= y^2 + 10y - 24 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad & t^2 + 9t + 8 \\ & = (t + 8)(t + 1) \end{aligned}$$

check:

$$\begin{aligned} (t + 8)(t + 1) &= t^2 + 8t + t + 8 \\ &= t^2 + 9t + 8 \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad & x^2 + 10x + 25 \\ & = (x + 5)(x + 5) \end{aligned}$$

check:

$$\begin{aligned} (x + 5)(x + 5) &= x^2 + 5x + 5x + 25 \\ &= x^2 + 10x + 25 \end{aligned}$$

$$\begin{aligned} \text{(e)} \quad & 2y^2 - 13y + 15 \\ & = 2y^2 - 10y - 3y + 15 \\ & = 2y(y - 5) - 3(y - 5) \\ & = (2y - 3)(y - 5) \end{aligned}$$

check:

$$\begin{aligned} (2y - 3)(y - 5) &= 2y^2 - 10y - 3y + 15 \\ &= 2y^2 - 13y + 15 \end{aligned}$$

$$\begin{aligned} \text{(f)} \quad & 16y^2 + 14y + 3 \\ & = 16y^2 + 8y + 6y + 3 \\ & = 8y(2y + 1) + 3(2y + 1) \\ & = (8y + 3)(2y + 1) \end{aligned}$$

check:

$$\begin{aligned} (8y + 3)(2y + 1) &= 16y^2 + 8y + 6y + 3 \\ &= 16y^2 + 14y + 3 \end{aligned}$$

Factor each of the following binomials, or state that they are prime (i.e. cannot be factored).
Check your answers by multiplying all the factors back together.

1. $x^2 - 9$

$$= (x-3)(x+3)$$

check:

$$\begin{aligned}(x-3)(x+3) &= x^2 + 3x - 3x - 9 \\ &= x^2 - 9\end{aligned}$$

2. $16 - 49w^2$

$$\begin{aligned}&= 4^2 - (7w)^2 \\ &= (4-7w)(4+7w)\end{aligned}$$

check:

$$\begin{aligned}(4-7w)(4+7w) &= 16 - 28w + 28w - 49w^2 \\ &= 16 - 49w^2\end{aligned}$$

3. $w^2 + 4$

prime

check:

4. $25u^2 + 1$

prime

check:

$$5. 4a^4 - 64$$

$$= (2a^2)^2 - 8^2$$

$$= (2a^2 - 8)(2a^2 + 8)$$

check:

$$(2a^2 - 8)(2a^2 + 8)$$

$$= 4a^4 - 16a^2 + 16a^2 - 64$$

$$= 4a^4 - 64$$

$$6. 8x^3 - 1$$

$$= (2x)^3 - 1^3$$

$$= (2x - 1)(4x^2 + 2x + 1)$$

check:

$$(2x - 1)(4x^2 + 2x + 1)$$

$$= 8x^3 + 4x^2 + 2x - 4x^2 - 2x - 1$$

$$= 8x^3 - 1$$

$$7. y^3 + 64$$

$$= y^3 + 4^3$$

$$= (y + 4)(y^2 - 4y + 16)$$

check:

$$(y + 4)(y^2 - 4y + 16)$$

$$= y^3 - 4y^2 + 16y + 4y^2 - 16y + 64$$

$$= y^3 + 64$$

$$8. 125c^3 + 27$$

$$= (5c)^3 + 3^3$$

$$= (5c + 3)(25c^2 - 15c + 9)$$

check:

$$(5c + 3)(25c^2 - 15c + 9)$$

$$= 125c^3 - 75c^2 + 45c + 75c^2 - 45c + 27$$

$$= 125c^3 + 27$$

$$9. x^3 - 1000$$

$$= x^3 - 10^3$$

$$= (x - 10)(x^2 + 10x + 100)$$

check:

$$(x - 10)(x^2 + 10x + 100)$$

$$= x^3 + 10x^2 + 100x - 10x^2 - 100x - 1000$$

$$= x^3 - 1000$$

Solve each of the following equations, using the Zero Product rule. Check your answers in the original equation.

1. $x^2 - 9 = 0$

$$(x-3)(x+3) = 0$$

$$x = 3, -3$$

check:

$$3^2 - 9 = 9 - 9 = 0$$

$$(-3)^2 - 9 = 9 - 9 = 0$$

2. $x^2 + 6x - 27 = 0$

$$(x+9)(x-3) = 0$$

$$x = -9, 3$$

check:

$$(-9)^2 - 6 \cdot 9 - 27$$

$$= 81 - 54 - 27 = 81 - 81 = 0$$

$$3^2 + 6 \cdot 3 - 27 = 9 + 18 - 27 = 0$$

3. $t^2 = t + 12$

$$t^2 - t - 12 = 0$$

$$(t-4)(t+3) = 0$$

$$t = 4, -3$$

check:

$$4^2 = 16 = 4 + 12$$

$$(-3)^2 = 9 = -3 + 12$$

$$4. 2x^2 - 15 = -x$$

$$2x^2 + x - 15 = 0$$

$$2x^2 + 6x - 5x - 15 = 0$$

$$2x(x+3) - 5(x+3) = 0$$

$$(2x-5)(x+3) = 0$$

$$x = \frac{5}{2}, -3$$

check:

$$2\left(\frac{5}{2}\right)^2 - 15 = \frac{25}{2} - \frac{30}{2} = -\frac{5}{2} \checkmark$$

$$2 \cdot 9 - 15 = 3 = -(-3) \checkmark$$

$$5. 10x^2 = 15x$$

$$10x^2 - 15x = 0$$

$$5x(2x-3) = 0$$

$$x = 0, \frac{3}{2}$$

check:

$$10 \cdot 0^2 = 0 = 15 \cdot 0 \checkmark$$

$$10 \cdot \frac{9}{4} = \frac{45}{2} = 15 \cdot \frac{3}{2} \checkmark$$

$$6. w^3 = 16w$$

$$w^3 - 16w = 0$$

$$w(w^2 - 16) = 0$$

$$w(w-4)(w+4) = 0$$

$$w = 0, 4, -4$$

check:

$$0^3 = 0 = 16 \cdot 0 \checkmark$$

$$4^3 = 64 = 16 \cdot 4 \checkmark$$

$$(-4)^3 = -64 = 16 \cdot (-4) \checkmark$$

$$7. x(6x+1) = 2$$

$$6x^2 + x - 2 = 0$$

$$6x^2 + 4x - 3x - 2 = 0$$

$$3x(2x+1) - 2(2x+1) = 0$$

$$(3x-2)(2x+1) = 0$$

$$x = -\frac{2}{3}, \frac{1}{2}$$

check:

$$-\frac{2}{3}(-\frac{2}{3}+1) = \frac{2}{9} \checkmark$$

$$\frac{1}{2}(3+1) = 2 \checkmark$$